A. Omitted Proofs

**Lemma A1:** The following inequality holds for any $x, y \in \mathbb{R}^+$, $n \geq 1$, and any $c_i, d_i \in [0, 1]$ $\forall i = 1 \ldots n$ satisfying $\sum_{i=1}^n c_i = \sum_{i=1}^n d_i = 1$ and at least one of $c_i$ or $d_i$ greater than zero $\forall i = 1 \ldots n$. Furthermore, it holds with equality if and only if $c_i = d_i \neq 0 \forall i = 1 \ldots n$.

$$\sum_{i=1}^n \frac{c_i d_i}{c_i x + d_i y} \leq \frac{1}{x + y}$$

**Proof of Lemma A1:** When $n = 1$, we must have $c_1 = d_1 = 1$ due to the constraints that each set of $c_i$ and $d_i$ must sum to one. In this case, the inequality is satisfied with equality. So, consider $n > 1$. I show that the left-hand side of the inequality reaches its global maximum of $\frac{1}{x + y}$ when $c_i = d_i \neq 0 \forall i = 1 \ldots n$ to show both that the inequality is always satisfied and that it only holds with equality at its peak.

I first show that $f(c_i, d_i) = \frac{c_i d_i}{c_i x + d_i y}$ is concave and use the fact that the sum of any number of concave functions is also concave. The second derivatives are

$$\frac{\partial^2 f(c_i, d_i)}{\partial c_i} = \frac{-2d_i^2 xy}{(c_i x + d_i y)^3} \leq 0, \quad \frac{\partial^2 f(c_i, d_i)}{\partial d_i} = \frac{-2c_i^2 xy}{(c_i x + d_i y)^3} \leq 0, \quad \text{and} \quad \frac{\partial^2 f(c_i, d_i)}{\partial c_i \partial d_i} = \frac{-2c_i d_i xy}{(c_i x + d_i y)^3}. \quad \text{Therefore,}$$

$$\frac{\partial^2 f(c_i, d_i)}{\partial c_i^2} \frac{\partial^2 f(c_i, d_i)}{\partial d_i^2} - \left(\frac{\partial^2 f(c_i, d_i)}{\partial c_i \partial d_i}\right)^2 = 0. \quad \text{Thus, the Hessian of } f(c_i, d_i) \text{ is negative semi-definite and therefore } f(c_i, d_i) \text{ is concave.}$$

If either $c_i$ or $d_i$ is equal to zero for some $i$, the corresponding term in the summation of the left-hand side is zero. So, consider the $n^* \leq n$ non-zero terms of the summation. We
must have $\sum_{i=1}^{n^*} c_i = v$ and $\sum_{i=1}^{n^*} d_i = w$ for some $v, w \leq 1$, due to the constraints. Consider the unconstrained maximization of the non-zero terms of the left-hand side of the inequality after using these constraints to substitute out $c_{n^*}$ and $d_{n^*}$. Because we are maximizing a concave function, the first-order conditions are necessary and sufficient for determining the global maxima of the function. We have, $\forall i = 1 \ldots n^* - 1$, the first-order conditions with respect to $c_i$:

$$
\frac{d^2 y}{(c_i x + d_i y)^2} = \frac{c_i^2}{(c_i x + d_i y)^2} = \frac{c_{n^*}^2}{(c_{n^*} x + d_{n^*} y)} = \frac{c_{n^*}}{d_{n^*}}
$$

The first-order conditions with respect to $d_i$ result in the same set of equations. Substituting for each $c_i$ in the constraint $\sum_{i=1}^{n^*} c_i = v$, we have $\frac{c_{n^*} x}{d_{n^*}} \sum_{i=1}^{n^*} d_i = v \iff c_{n^*} = d_{n^*} \frac{v}{w}$ which then implies $c_i = d_i \frac{v}{w} \forall i = 1 \ldots n^*$. Using this relationship, we have $\sum_{i=1}^{n^*} c_i d_i = \frac{v w}{w x + w y} \leq \frac{1}{x+y}$, where the inequality holds with equality if and only if $v = w = 1$ which implies $c_i = d_i \neq 0 \forall i = 1 \ldots n$.

**Lemma A2:** $\sum_{e_i \in E} e_i | V = 0 e_i | V = 1 \left( e_i | V = 1 - e_i | V = 0 \right)$ has the same sign as $(1 - 2p_i)$, provided one of the event realizations is informative.

**Proof of Lemma A2:** Using the assumed symmetry of the public events, $e_i | V = 0 = e_i | V = 1$ and $e_i | V = 1 = e_i | V = 0$ for events $i$ and $j$, we can write this summation as a sum over only the 'positive' events, $E \equiv \{ e_i \in E | Pr(e = e_i | V = 1) > \frac{1}{2} \}$. Letting $e^j$ be the symmetric counterpart to the positive event, $e^i$, we have

$$
\sum_{e_i \in E} \frac{e_i^b | V = 0 e_i^b | V = 1 (e_i^b | V = 1 - e_i^b | V = 0)}{Pr(e = e^j)^2} = \sum_{e_i \in E} \frac{e_i^b | V = 0 e_i^b | V = 1 (e_i^b | V = 1 - e_i^b | V = 0)}{Pr(e = e^j)^2} \frac{1}{Pr(e = e^j)^2 - Pr(e = e^j)^2} = \sum_{e_i \in E} e_i^j | V = 0 e_i^j | V = 1 (e_i^j | V = 1 - e_i^j | V = 0) \frac{1}{Pr(e = e^j)^2 - Pr(e = e^j)^2}
$$

Now, $Pr(e = e^j)^2 - Pr(e = e^j)^2 = \left( p_0 e_i^j | V = 1 + (1 - p_0) e_i^j | V = 0 \right)^2 - \left( p_0 e_i^j | V = 1 + (1 - p_0) e_i^j | V = 0 \right)^2 = (e_i^j | V = 1 - e_i^j | V = 0)^2 (1 - 2p_i)$ so that

$$
\sum_{e_i \in E} e_i^j | V = 0 e_i^j | V = 1 (e_i^j | V = 1 - e_i^j | V = 0) \frac{1}{Pr(e = e^j)^2 - Pr(e = e^j)^2} = (1 - 2p_i) \sum_{e_i \in E} e_i^j | V = 0 e_i^j | V = 1 \frac{(e_i^j | V = 1 - e_i^j | V = 0)^3}{Pr(e = e^j)^2 Pr(e = e^j)^2}
$$

\[1\]Any uninformative event such that $e_i^j | V = 1 = e_i^j | V = 0$ contributes zero to the summation.
The summation in (1) is strictly positive if one of the event realizations is informative \((Pr(e = e^i | V = 1) \neq Pr(e = e^i | V = 0))\) so that the sign of \(\sum_{e^i \in E} e^i | V = 0 \frac{(e^i | V = 1 - e^i | V = 0)}{Pr(e = e^i)^2}\) is the same as the sign of \((1 - 2p_0)\). □

**Lemma A3:** A trader’s first period expected profit is strictly increasing in her first period signal quality, \(\frac{\partial \pi_0(\beta, p_0, q_0)}{\partial q_0} > 0\).

**Proof of Lemma A3:**

We have

\[
\frac{\partial \pi_0(\beta, p_0, q_0)}{\partial q_0} = 2\omega_0 m \left( \frac{1}{Pr(a_0 = buy)} + \frac{1}{Pr(a_0 = sell)} \right) - \omega_0 m (2q_0 - 1) \mu \beta (2p_0 - 1) \left( \frac{1}{Pr(a_0 = buy)^2} - \frac{1}{Pr(a_0 = sell)^2} \right)
\]

Consider the sign of the term corresponding to a purchase:

\[
2\frac{Pr(a_0 = buy)}{Pr(a_0 = sell)} - (2q_0 - 1) \mu \beta (2p_0 - 1) \frac{1}{Pr(a_0 = buy)^2}
\]

\[
= \frac{2Pr(a_0 = buy) - (2q_0 - 1) \mu \beta (2p_0 - 1)}{Pr(a_1 = buy)^2}
\]

\[
= \frac{\mu \beta (2p_0 q_0 + 2(1-p_0)(1-q_0) - (2q_0 - 1)(2p_0 - 1)) + 2m}{Pr(a_1 = buy)^2}
\]

\[
= \frac{\mu \beta + 2m}{Pr(a_1 = buy)^2}
\]

\(> 0\)

Similarly, the sign of the term corresponding to a sale is strictly greater than zero, so that overall, \(\frac{\partial \pi_0(\beta, p_0, q_0)}{\partial q_0} > 0\). □

**B. Robustness**

**B.1 Multiple Signals**

To motivate the assumption that a trader must choose which signal to acquire, consider an example: the recent announcement that Walt Disney Company made a deal to purchase the majority of the assets of 21st Century Fox.\(^2\) Upon hearing the announcement, many questions come to mind, all of which affect the value of the deal and hence the value of Disney.\(^3\) A trader could try to find quick answers to these questions, giving noisy signals of the overall value of the merger (i.e. through internet searches). Alternatively, she could call an analyst or someone within the company’s management. Getting ahold of someone in the know would take longer, but almost certainly produce a better overall signal of the merger’s

\(^2\)The deal was announced Dec. 14, 2017 and is valued at $52.4 billion. https://www.nytimes.com/2017/12/14/business/dealbook/disney-fox-deal.html

\(^3\)Examples include the following. Which assets are being transferred? Are there cost savings generated by the merger? How does the deal affect Disney’s ability to launch their own streaming channel? Will the deal be shut down by antitrust concerns?
value. Importantly, both pursuits take time so cannot be done simultaneously.

The model predicts that competition induces traders to rely on rapid, noisy sources for answers rather than placing a call. By why not make the call after obtaining an answer to one of the many questions? Certainly, it’s possible to do so, but if there are many other questions that could be answered quickly, the trader may consider seeking another answer rather than making the call. Conceptually, we can think about extending the model to more than two periods such that, in each period the trader can acquire a weak signal immediately or a better signal with delay. In each period, the trader would then face a trade-off similar to the two-period model.

Extending the model to multiple periods is non-trivial, however, because additional forces come into play. First, traders possess residual private information not revealed by past trades, and their decision to obtain a weak or strong signal can be conditioned on this information, enlarging the strategy space. Second, position limits may alter the value of additional information (i.e. the value of information is likely to be different if a trader can only unwind her initial position). Third, with many periods to trade over, a trader will spread out her trades to hide her information (as in Kyle (1985)). Finally, strategic incentives to manipulate prices in order to profit in the future may exist (Chakraborty and Yilmaz, 2004). Thus, although the intuitive nature of a time cost extends to more than two periods, future work requires a tractable model to deal with these issues.

A setting in which the model can be directly applied is one of an irreversible investment. Consider a venture capital firm considering an investment in a startup (for simplicity, assume no private information on either side). The cost of pursuing a lengthy evaluation process is that public information may arrive that reveals information about the startup’s value to both sides, which in expectation raises the price when the investment is worthwhile. The venture capital firm may therefore prefer to condition it’s decision on more readily available, albeit noisier, information. In this example, a ‘trade’ is one-sided in that the venture capital firm can only purchase equity (or not). However, I show in Part C of this Online Appendix that the result of Proposition 2 continues to apply when only one-sided trades are possible.

Contrary to the assumption, we could think of examples where only a single source of information exists. If research produces a stream of signals and a trader can costlessly adjust her position as each new signal arrives, then clearly she would have the incentive to do so and there would be no trade-off between weaker and better information. In many real-world situations, however, multiple sources of information exist.
Figure 1: Threshold Probability of Informed Trading

Note: Contour plots of the upper limit on the probability of informed trading, $\mu_A = \mu_B$, required to guarantee that the unique equilibrium is one in which informed traders trade in the period they obtain information (left graph), and to guarantee that the probability that the first trader rushes peaks at $p_0 = \frac{1}{2}$ (right graph). The relevant parameter space lies below the diagonal. For the left graph, I plot the minimum value of $\mu_A = \mu_B$ over the whole range of the prior, $p_0 \in (0, 1)$. The contour lines in both graphs are in increments of 0.1 with the lowest corresponding to $\mu_A = \mu_B = 1$.

B.2 Limit on the Probability of Informed Trading

The analytical results in the paper rely on the probability of informed trading, $\mu$, being sufficiently small. However, for most parameterizations, no upper limit on $\mu$ is in fact required, and for the remaining parameterizations, the upper limit is large. To illustrate, Figure 1 provides contour plots of the numerically calculated upper limit as a function of the two signal strengths for the competitive model of Section 3. The left-hand graph provides the upper limit required to guarantee the unique equilibrium is one in which informed traders trade in the period they obtain information (Proposition 4, part a) and the right-hand graph provides that required to guarantee that the probability that the first trader rushes peaks at $p_0 = \frac{1}{2}$ (Proposition 4, part d).

The left graph of Figure 1 shows that the limit required to ensure equilibrium uniqueness only binds in the extreme case in which both signal qualities approach one. From the right graph, we see that when both signal qualities exceed about 0.75, an upper limit needed to guarantee the time cost is largest at maximum uncertainty is necessary: the combination of a high probability of informed trade and strong signal qualities makes the effect of the bid-ask spread more pronounced. To put the required limit on the probability of informed trade into perspective, the most conservative estimate of it which I’m aware is that obtained by Cipriani and Guarino (2014). They estimate $\mu = 0.42$ where their model defines $\mu$ to be conditional on information being available, and allows for informed traders to have imperfect
Comparing their estimate with $\mu_A = \mu_B$ in Figure 4, we see that the result continues to hold except in the very extreme case of signal qualities that approach one. One could argue instead that, if information is only available on news days (as in the models of Cipriani and Guarino (2014) and Easley et al. (1996)), and that the marker maker can’t condition on news being available or not, then the unconditional probability of informed trading is the correct measure for comparison. Cipriani and Guarino (2014) estimate this probability to be 0.19. At this value, the result holds for all parameterizations of the model, because the minimum upper limit is 0.38.

When the probability of informed trading becomes large, the informed trader may no longer find it optimal to trade immediately after rushing, instead preferring to delay her trade to $t = 1$ (or to mix between the two periods). Intuitively, when the cost of waiting is large because the other trader’s trade reveals a lot of information, informed trade is concentrated at $t = 0$, making the spread large. In this case, after acquiring a signal at $t = 0$, it may be optimal to delay trading until $t = 1$ where the spread is smaller. An equilibrium in this case requires a careful construction of mixing probabilities. The informed trader must be indifferent between rushing and waiting, and must also be indifferent after rushing, between trading immediately and delaying her trade to the second period, for both realizations of her private signal.

The result that a trader rushes most often at maximum uncertainty (Propositions 2 and 4) requires the probability of informed trading to not be too large for a slightly different reason. When the probability of informed trade is small, because (by Lemma 2) the expected profit in each period is concave and approaches zero as public beliefs become extreme, as we move towards $p_0 = \frac{1}{2}$, the expected profit from waiting increases more than that from rushing, causing the trader to wait more often (in the absence of any additional cost beyond the trading cost). In this case, we have the standard intuition that the marginal value of better information is highest at high uncertainty. However, when the probability of informed trading is large, trades in the second period can be very informative if trade is (endogenously) concentrated there. In this case, as we move towards $p_0 = \frac{1}{2}$, is it possible that the expected profit from waiting actually increases less than that of the expected profit from rushing so that a trader may actually rush more often. The marginal value of information is distorted by the large trading costs in this case. In fact, the value of information (Proposition 1, part b)) no longer peaks at $p_0 = \frac{1}{2}$, instead reaching local maxima at two values of $p_0$ located symmetrically around $\frac{1}{2}$. Correspondingly, the informed trader’s rushing probability is no longer a maximum at $p_0 = \frac{1}{2}$ (Proposition 2, part a) and Proposition 4, part d)), instead

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4The traditional probability of informed trading measure, PIN, of Easley et al. (1996), assumes instead that informed trades are perfectly informed, resulting in lower probability of informed trading estimates.
similarly reaching local maxima at values of $p_0$ located symmetrically around $\frac{1}{2}$.

Therefore, a small probability of informed trading is not only sufficient, but also necessary for the main results of the paper to hold.

### B.3 Alternative Asset Value Distributions and Information Structures

The results in Propositions 2 and 4 that traders acquire less information when the prior uncertainty is highest is a result of two generic features of the model. First, expected profits increase with uncertainty. Second, others’ trades reduce uncertainty in the asset value when one waits, and more so when uncertainty is higher. Together these features imply the largest decrease in the expected profit from waiting when uncertainty is high, which in turn encourages more rushing. Here, I show that these two features are also present in a model à la Kyle (1985) in which the asset value is normally distributed, suggesting that the results are not driven by the binary nature of the asset’s value.

Consider a version of the model of Kyle (1985) in which a perfectly informed, risk-neutral informed trader trades with a risk-neutral market maker. I assume there are two trading periods, and that the informed trader must choose between a weak signal in the first period and a strong signal in the second period. To make the analysis as simple as possible, I assume the trader receives no information in the first period but perfect information if she waits. Obviously this assumption oversimplifies the problem, so that the trader faces no trade-off, but it is sufficient to demonstrate the nature of the time cost.

The informed trader can trade any quantity, and is strategic, accounting for the price impact of her trade. Let the asset value be distributed normally, $V \sim N(0, \sigma^2)$. The public signal between trading periods is given by $s_p = V + \varepsilon$ where the noise is also distributed normally, $\varepsilon \sim N(0, \sigma^2_p)$, and is independent of $V$. Finally, there is noise trader demand in the second period given by $u \sim N(0, \sigma^2_u)$.

Because $s_p$ and $V$ are jointly normally distributed, the updated public belief after observing the public signal is given by $p_1 = E[V|s_p] = \frac{\sigma^2}{\sigma^2 + \sigma^2_p}s_p$. The conditional variance of the asset value at this time is given by $Var(V|s_p) \equiv \Sigma_1 = \frac{\sigma^2}{\sigma^2 + \sigma^2_p}$. Kyle (1985), Theorem 1, shows that there exists a linear equilibrium in which the informed trader, knowing $V$,

---

5 It remains true that as the initial price becomes very certain, although the value of information goes to zero, the probability with which the informed trader rushes also approaches zero (i.e. she waits with probability approaching one).

6 Note that no trade occurs in the first period because the informed trader has no information at this time if she waits.
submits demand, \( x = b(V - p_1) \), and the market maker sets a price after observing demand, 
\( p_2 = p_1 + \lambda(x + u) \) where \( b = \left( \frac{\sigma^2}{\sigma^2 + \sigma_p^2} \right)^{\frac{1}{2}} \) and \( \lambda = \frac{1}{2} \left( \frac{\sigma^2}{\sigma_p^2} \right)^{\frac{1}{2}}. \) The informed trader’s expected profit conditional on observing the public signal and \( V \) (where the expectation is over the uninformed demand) is then given by

\[
\pi_1|V, s_p = E \left[ x(V - p_2) \right] = x(V - p_1 - \lambda x) = b(V - p_1)^2 - \lambda b^2 (V - p_1)^2 = b(1 - \lambda b)(V - p_1)^2 = \frac{b}{2}(V - p_1)^2
\]

using \( b\lambda = \frac{1}{2}. \) When considering whether or not to wait, the informed trader’s ex ante profit is then given by the expectation of (2) over \( s_p \) and \( V \):

\[
\pi_1 = E \left[ \frac{b}{2}(V - p_1)^2 \right] = E \left[ \frac{b}{2} \left( V - \frac{\sigma^2}{\sigma^2 + \sigma_p^2} (V + \varepsilon) \right)^2 \right] = \frac{b}{2} \left[ E \left( \frac{\sigma^2}{\sigma^2 + \sigma_p^2} V \right)^2 + E \left( \frac{\sigma^2}{\sigma^2 + \sigma_p^2} \varepsilon \right)^2 \right] = \frac{1}{2} \sigma_u \left( \frac{\sigma^2}{\sigma^2 + \sigma_p^2} \right)^{-\frac{1}{2}} \left[ \left( \frac{\sigma^2}{\sigma^2 + \sigma_p^2} \right)^2 \sigma^2 + \left( \frac{\sigma^2}{\sigma^2 + \sigma_p^2} \right)^2 \sigma_p^2 \right] = \frac{1}{2} \left( \frac{\sigma^2}{\sigma^2 + \sigma_p^2} \right)^{\frac{1}{2}} \left( \frac{\sigma^2}{\sigma^2 + \sigma_p^2} \right)^{\frac{1}{2}}
\]

where the second last equality uses the fact that \( V \) and \( \varepsilon \) are independent and mean zero so that \( E[V\varepsilon] = 0. \) (3) is increasing in the prior variance in the asset value, \( \sigma^2: \)

\[
\frac{\partial \pi_1}{\partial \sigma^2} = \frac{1}{4} \left( \frac{\sigma^2 \sigma^2 \sigma_p^2}{\sigma^2 + \sigma_p^2} \right)^{-\frac{1}{2}} \left( \frac{\sigma^2 \sigma_p^2 (\sigma^2 + \sigma_p^2) - \sigma^2 \sigma_p^2}{(\sigma^2 + \sigma_p^2)^2} \right) = \frac{1}{4} \left( \frac{\sigma^2 \sigma^2 \sigma_p^2}{\sigma^2 + \sigma_p^2} \right)^{-\frac{1}{2}} \left( \frac{\sigma^2 \sigma_p^2}{\sigma^2 + \sigma_p^2} \right) > 0
\]

In the absence of a public signal \( (\sigma_p^2 \to \infty) \), we have

\[
\pi_1^{NP} = \frac{1}{2} \left( \frac{\sigma^2}{\sigma^2 + \sigma_p^2} \right)^{\frac{1}{2}} \]

Comparing (3) with (4), we see that the public signal reduces the ex ante profit by a factor of \( \left( \frac{\sigma^2}{\sigma^2 + \sigma_p^2} \right)^{\frac{1}{2}} < 1. \) Furthermore, as uncertainty in the asset value, \( \sigma^2 \), increases, this reduction factor gets smaller, approaching zero. Therefore, the public signal reduces

\footnote{The formula for \( \lambda \) in the statement of Theorem 1 in Kyle (1985) contains a typo.}
the expected profit by a larger amount when uncertainty is higher, so that the time cost increases with uncertainty as with a binary asset value.

B.4 Multiple Trades

To simplify the exposition, I restricted the informed trader to a single trade if she rushes. Here, I argue that the main insight of the model is unlikely to change if she could acquire (or sell short) an additional unit in the second period, or instead unwind her position. In the latter case, the equilibrium is unchanged because, given that she doesn’t receive any additional private information and that her information is not fully revealed by her trade, she still has a private belief more extreme than the market maker, so prefers to hold her position.

On the other hand, because the trader has residual private information after trading, she would like to acquire (or sell) an additional unit in the second period if she could, earning a small additional profit and revealing more of her private signal to the market. Therefore, if I were to allow a second trade, it would make rushing (weakly) more attractive which I conjecture can only lead to more rushing in equilibrium. Certainly, by changing the profit to rushing, this variation changes the equilibrium probability of acquiring better information, but it doesn’t change the nature of the impact of competition on second period trading profits, and therefore is unlikely to change the main message of the paper.

B.5 The Role of the Bid-Ask Spread

In the model, the market maker can post different prices in each period, conditional on the quality of information available in each period. In certain situations, such as after a news release, this knowledge of a 'time zero' is natural, but in other situations, it may be more natural to assume that the market maker instead only knows the average quality of information (i.e. he knows some traders have weak information and some have strong, but both could be present at any time). With this assumption, I suspect the main results of the paper continue to hold for two reasons. The first is that I obtain results for small values of the probability of informed trading where the bid-ask spread is close to zero in each period, so that it plays little role. Second, it is possible to write down a model in which the market maker posts only a single price equal to the expected value of the asset. In this case, I obtain

---

8In fact, this property of the model is shared by the original Glosten and Milgrom (1985) model and the subsequent literature.

9The profit from rushing only weakly increases because, if the bid-ask spread in the second period is large enough, the second trade is not profitable.
a result very similar to Proposition 2 - better information is forgone at $p_0 = \frac{1}{2}$ when a trader would be most willing to pay a monetary cost for it. Therefore, the precise way the bid-ask spread is determined is not important for the results. Additionally, neither is the fact that the trader has market power, because in the single price version of the model, she doesn’t affect the price until after she trades.

C. One-sided Trades

I consider a variation of the single trader model in which she can only trade in one direction as would be the case with short-selling restrictions or when making a one-time investment. I explicitly study the case in which only purchases are possible, but the results are the same if only sales are possible. The only other modification to the model is to assume that the uninformed traders buy in each period with equal probability. I maintain the assumption that the public event may reveal positive or negative information.

In this version of the model, a version of Lemma 1 continues to hold: traders only buy when they have positive signals. We can then easily show that the expected profits in each period are given by (re-using the notation of the model in the paper):

$$\pi_0(\beta, p_0, q_0) = \frac{\omega_0 m (2q_0 - 1)}{Pr(a_0 = buy)}$$

and

$$\pi_1(\beta, p_0, q_1) = \omega_0 m (2q_1 - 1) \sum_{e' \in E} \frac{e_{V=0}^{e_i} e_{V=1}^{e_i}}{Pr(a_1 = buy, e = e_i)}$$

where $m = \frac{1-\mu}{2}$. Lemma 2 no longer holds because symmetry around $p_0 = \frac{1}{2}$ is broken: positive signals are more valuable when the prior suggests the asset is worth zero ($p_0 < \frac{1}{2}$), and conversely. However, the main result, Proposition 2, holds as proven below.

Proof of Proposition 2 with One-sided Trades:

Part a). The proof that, for a sufficiently small probability of informed trading ($\mu \leq \bar{\mu}_{21}$ for some $\bar{\mu}_{21} > 0$), the informed trader strictly prefers to trade in the first period after rushing is identical to that in the original proof of Proposition 2. From the calculations in the proof of Proposition 1, it is also clear that $\frac{\partial \pi_0(\beta, p_0, q_0)}{\partial \beta} < 0$, $\frac{\partial \pi_1^{NP}(\beta, p_0, q_1)}{\partial \beta} > 0$, and $\frac{\partial \pi_1(\beta, p_0, q_1)}{\partial \beta} > 0$ continue to hold. Therefore, as in the original proof, $\beta^*$, is unique.
Part b). Assume \( \mu \leq \bar{\mu}_{21} \) so that an equilibrium in which each trader trades in the period she acquires information exists (by part a)). As in the original proof, if \( \beta^* \) is interior over some range of \( p_0 \), the implicit function theorem gives:

\[
\frac{d\beta^*}{dp_0} = \frac{\partial \pi_0(\beta^*, p_0, q_0)}{\partial p_0} - \frac{\partial \pi_1(\beta^*, p_0, q_1)}{\partial \beta^*} - \frac{\partial \pi_0(\beta^*, p_0, q_0)}{\partial \beta^*} (6)
\]

From part a), the denominator of (6) is strictly positive. For the terms in the numerator, we have

\[
\frac{\partial \pi_0(\beta^*, p_0, q_0)}{\partial p_0} = \frac{(1 - 2p_0)}{\omega_0} \pi_0(\beta^*, p_0, q_0) - \omega_0 m(2q_0 - 1) \frac{buy_{0|V_1} - buy_{0|V_0}}{Pr(a_0 = buy)^2}
\]

and

\[
\frac{\partial \pi_1(\beta^*, p_0, q_1)}{\partial p_0} = \frac{(1 - 2p_0)}{\omega_0} \pi_1(\beta^*, p_0, q_1) + \Sigma
\]

where

\[
\Sigma = -\omega_0 m(2q_1 - 1) \sum_{e^i \in E} e^i_{V=0} e^i_{V=1} \frac{\partial}{\partial p_0} (Pr(a_1 = buy, e = e^i)) \sum_{e^i \in E} e^i_{V=0} e^i_{V=1} \frac{Pr(a_1 = buy, e = e^i)^2}{Pr(a_1 = buy)^2}
\]

The numerator of (6) is then

\[
\frac{\partial \pi_0(\beta^*, p_0, q_0)}{\partial p_0} - \frac{\partial \pi_1(\beta^*, p_0, q_1)}{\partial p_0} = \frac{(1 - 2p_0)}{\omega_0} (\pi_0(\beta^*, p_0, q_0) - \pi_1(\beta^*, p_0, q_1)) - \omega_0 m(2q_0 - 1) \frac{buy_{0|V_1} - buy_{0|V_0}}{Pr(a_0 = buy)^2} - \Sigma
\]

using the fact that, when \( \beta^* \) is interior, \( \pi_0(\beta^*, p_0, q_0) = \pi_1(\beta^*, p_0, q_1) \). In the limit as \( \mu \to 0 \), the first term approaches zero so that the sign of \( \frac{d\beta^*}{dp_0} \) is then determined by

\[
-\lim_{\mu \to 0} \Sigma = \omega_0 (2q_1 - 1) \sum_{e^i \in E} e^i_{V=0} e^i_{V=1} \frac{(e^i_{V=1} - e^i_{V=0})}{Pr(e=e^i)^2}
\]

From Lemma A2, \( \sum_{e^i \in E} e^i_{V=0} e^i_{V=1} \frac{(e^i_{V=1} - e^i_{V=0})}{Pr(e=e^i)^2} \) has the same sign as \( (1 - 2p_0) \), so that,
in the limit, $\frac{d\beta^*}{dp_0}$ is strictly increasing for $p_0 < \frac{1}{2}$, zero at $p_0 = \frac{1}{2}$, and strictly decreasing for $p_0 > \frac{1}{2}$. By continuity, because these properties of $\frac{d\beta^*}{dp_0}$ are strict, there exists a $\bar{\mu}_{22} > 0$ such that for all $\mu \leq \bar{\mu}_{22}$, $\frac{d\beta^*}{dp_0}$ peaks at $p_0 = \frac{1}{2}$.

The remainder of the proof, for the case in which $\beta^*$ is not interior, is identical to the original proof. □